

MATH-330

Martingales and Brownian motion

Blanc Guillaume

Cursus	Sem.	Type
Mathematics	BA6	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Written
Workload	150h
Weeks	14
Hours	4 weekly
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Summary

Introduction to the theory of discrete-time martingales (optional stopping and convergence theorems) and its applications (gambler's ruin, branching processes, Pólya urns...). Introduction to Brownian motion.

Content

Martingales are stochastic processes with a very natural special property which enables probabilists to prove a lot of theorems. They play a fundamental role in Probability theory, as well as in its applications to other fields (e.g. finance, population modelling, statistics, ...).

The course will cover the following topics:

- conditional expectation
- discrete-time stochastic processes and martingales,
- stopping times and optional stopping theorem,
- convergence of martingales,
- applications: gambler's ruin, branching processes, Pólya urns,
- introduction to Brownian motion,
- basic properties of Brownian motion.

Keywords

martingales, optional stopping theorem, martingale convergence theorem, applications of martingales, Brownian motion

Learning Prerequisites**Required courses**

First and second year courses of the mathematics bachelor program

Recommended courses

Probability theory (MATH-432), Measures and integration (MATH-303)

Teaching methods

Blackboard lectures and exercise sessions

Expected student activities

Attending the lectures and working on the exercise sheets

Assessment methods

Written exam

Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés.

Supervision

Office hours	No
Assistant.e.s	Yes
Forum	Yes

Resources

Bibliography

"Measure Theory, Probability, and Stochastic Processes", Jean-François Le Gall, Springer (2022)

"Probability: Theory and Examples", Rick Durrett, Cambridge University Press (5th ed, 2019)

"Brownian motion", Peter Mörters and Yuval Peres, Cambridge University Press (2010)

"Brownian Motion, Martingales, and Stochastic Calculus", Jean-François Le Gall, Springer (2016)

Ressources en bibliothèque

- [Find the references at the Library](#)

Notes/Handbook

Some lecture notes will be made available during the semester

Moodle Link

- <https://go.epfl.ch/MATH-330>

Prerequisite for