EPFL

MATH-564

# Convex geometric analysis

Tiba Marius

| Cursus | Sem. | Type |
|---|---|---|
| Ing.-math | MA2, MA4 | Opt. |
| Mathématicien | MA2 | Opt. |

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| Language of teaching | English |
| Credits | 5 |
| Session | Summer |
| Semester | Spring |
| Exam | Written |
| Workload | 150h |
| Weeks | 14 |
| **Hours** | **4 weekly** |
| Courses | 2 weekly |
| Exercises | 2 weekly |
| **Number of positions** | |


## Summary

In this course we study the fundamental notion of convexity at the interface of analysis and geometry, an area that has seen remarkable progress in recent years.


## Content

The starting point of this course is the classical isoperimetric inequality, dating back to antiquity: in the plane, of all shapes of a given perimeter, the disc has the largest area. Isoperimetric inequalities are prevalent in many familiar settings e.g. Euclidean and Gaussian spaces and the sphere, and they give rise to the surprising concentration of measure phenomenon: a Lipschitz function on a high dimensional sphere or Gaussian space concentrates around its mean. These are part of a large family of important geometric and functional inequalities including Brunn-Minkowski, Prekopa-Leindler and Sobolev, with applications to other fields such as PDEs. An active research topic is the investigation of equality or near equality cases of these inequalities. We will apply several techniques to study these inequalities: symmetrization processes, convex localization and optimal transport.

The second part of the course focuses on the distribution of mass in high dimensional convex bodies. We will mention two recent breakthroughs: the resolution of Bourgain's slicing conjecture - every convex body of unit volume has a slice with constant volume, and of the thin shell conjecture. We will derive a central limit theorem for convex bodies: in most directions the distribution of mass is approximately Gaussian. We will prove Dvoretzky's celebrated theorem: all Banach spaces have an almost Euclidean subspace of high dimension; equivalently, all convex bodies have almost ellipsoidal sections of quite high dimension. We will explore the notion of convex duality culminating with Blaschke-Santalo and Bourgain-Milman inequalities. Throughout, we will develop probabilistic techniques.

## Learning Prerequisites

### Required courses

- Analysis I - Real Analysis
- Analysis II - Vector Analysis
- Probability (Introductitiory)

The first exercise class will be devoted to recalling the necessary background

### Recommended courses

- Analysis IV - Lebesgue measure, Fourier analysis

### Important concepts to start the course

- Multivariable Calculus (Differentiation and Integration in R^n, Volume and Surface area formulae)
- Basic Concepts in Probability (Probability Density Function, Expectation, Independence, Central Liimit Theorem)

## Learning Outcomes

By the end of the course, the student must be able to:

- Prove classical geometric and functional inequalities
- Investigate the equality cases (or stability) of these inequalities
- Use techniques such as symmetrization, convex localization and optimal transport
- Apply these inequalities to other contexts
- Quantify high-dimensional phenomena arising from convexity
- Compute concrete examples
- Prove main results

## Teaching methods

Weekly lectures and exercise sessions with assistant

## Expected student activities

Students are expected to attend the lectures and actively participate in the exercise sessions. In addition, they are expected to attempt the problems on the exercise sheets and to submit their solutions of a selected subset of the exercises for grading.

## Assessment methods

Weekly graded homeworks and written final exam

## Supervision

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| Office hours | Yes |
| Assistant.e.s | Yes |
| Forum | Yes |

## Resources

### Bibliography


- K. Böröczky, A. Figalli, J. Ramos. Isoperimetric inequalities, Brunn-Minkowski theory and Minkowski type Monge-Ampère equations on the sphere. EMS
- R. Gardner. The Brunn-Minkowski Inequality. Bull. of the AMS, 2002
- S. Artstein-Avidan, A. Giannopoulos, V. Milman, Asymptotic Geometric Analysis, part I. Mathematical Surveys and Monographs 202 AMS, 2015
- S. Brazitikos, A. Giannopoulos, P. Valettas, B.-H. Vritsiou, Geometry of Isotropic Convex Bodies. Mathematical Surveys and Monographs 196 AMS, 2014


### Notes/Handbook

Lecture notes will be provided

### Moodle Link

- https://go.epfl.ch/MATH-564