MATH-318

Set theory

Duparc Jacques

Cursus	Sem.	Type
Ing.-math	MA2, MA4	Opt.
Mathématicien	MA2	Opt.

Language of teaching	English
Credits	5
Session	Summer
Semester	Spring
Exam	Written
Workload	150h
Weeks	14
Hours	**4 weekly**
Courses	2 weekly
Exercises	2 weekly
Number of positions	

Summary

Set Theory as a foundational system for mathematics. ZF, ZFC and ZF with Atoms. Relative consistency of the Axiom of Choice, the Continuum Hypothesis, the reals as a countable union of countable sets, the existence of a countable family of pairs without any choice function, and so on.

Content

Part I: The Theory Itself:

1.The Axioms
2. Well-orderings and Ordinals
3. Extension by Definition and Conservative Extension
4. axiom of Choice and Cardinals
5. Well-Founded Sets and the Axiom of Foundation

Part II: Relativization and Absoluteness

6 From Inside a Class (Relativization)
7 The Mostowski Collapse
8 Preservation under Relativization andÂ Absoluteness

Part III: The Consistency of ZF

9 Arithmetic and Recursivity
10 Jech's Proof of Gödel's Second Incompleteness Theorem for Set Theory

Part IV: Gödel's Constructible Universe

11 The Constructible Sets 1
12 AC and CH inside Gödel's Constructible Universe

Part V: Forcing

13 Forcing Conditions and Generic Filters
14 P-names and Generic Extensions
15 The Truth Lemma
16 ZFC within the Generic Extension and Cardinal Preservation
17 Independence of CH
18 Independence of AC

Part VI: ZF without the Axiom of Choice

19 Cardinality Revisited
20 About R without the axiom of choice (Outcomes of the reals as a Countable Union of Countable Sets)
21 Symmetric Submodels of Generic Extensions
22 Some Applications of the Symmetric Submodel Technique
22.1 Forcing the reals as a Countable Union of Countable Sets
22.2 Forcing the Well-Orderings of the Reals Out
22.3 Forcing Every Ultrafilter on Ï# is Principal

Part VII: Set Theory with Atoms

23 Atoms and Permutation Models
23.1 Zermelo-Fraenkel with Atoms (ZFA)
23.2 Permutation Models
23.3 The Basic Fraenkel Model
23.4 The Second Fraenkel Model
23.5 The Ordered Mostowski Model
24 Simulating Permutation Models by Symmetric Models
24.1 The Jech-Sochor Embedding Theorem
24.2 Some applications of the Jech-Sochor Embedding Theorem

Keywords

Set Theory, Relative consistency, ZFC, Ordinals, Cardinals, Transfinite recursion, Relativization, Absoluteness, Constructible universe, L, Axiom of Choice, Continuum hypothesis, Forcing, Generic extensions

Learning Prerequisites

Required courses

MATH-381 Mathematical Logic (or any equivalent course).
In particular ordinal and cardinal numbers, ordinal and cardinal arithmetic will be considered known and admitted.

Recommended courses

Mathematical logic (or any equivalent course on first order logic). Warning: without a good understanding of first order logic, students tend to get definitely lost sooner or later.

Important concepts to start the course

- 1st order logic
- ordinal and cardinal arithmetics
- elements of proof theory
- very basic knowledge of model theory
- the compactness theorem
- Löwenheim-Skolem theorem
- the completeness theorem for 1st order logic

Learning Outcomes

By the end of the course, the student must be able to:

- Specify a model of ZFC
- Prove consistency results
- Develop a generic extension

- Argue by transfinite induction
- Decide whether ZFC proves its own consistency
- Formalize the axioms of ZF, AC, CH, DC
- Sketch an inner model
- Justify the axiom of foundation
- Formalize a model in which the reals are a countable union of countable sets
- Produce a model in which a countable set of pairs has no choice function
- Create a model in which the finite subsets of an infinite set is mapped onto the set of all its subsets

Teaching methods

Ex cathedra lecture and exercises

Expected student activities

- Attendance at lectures
- Solve the exercises

Assessment methods

- Writen exam (3 hours).
- Dans le cas de l'art. 3 al. 5 du Règlement de section, l'enseignant décide de la forme de l'examen qu'il communique aux étudiants concernés

Supervision

Office hours	No
Assistant.e.s	Yes
Forum	Yes

Resources

Virtual desktop infrastructure (VDI)

No

Bibliography

1. Kenneth Kunen: Set theory, Springer, 1983
2. Lorenz Halbeisen: Combinatorial Set Theory, Springer 2018
3. Thomas Jech: Set theory, Springer 2006
4. Jean-Louis Krivine: Theorie des ensembles, 2007
5. Patrick Dehornoy: Logique et théorie des ensembles; Notes de cours, FIMFA ENS: http://www.math.unicaen.fr/~dehornoy/surveys.html
6. Yiannis Moschovakis: Notes on set theory, Springer 2006
7. Karel Hrbacek and Thomas Jech: Introduction to Set theory, (3d edition), 1999

Ressources en bibliothèque

- [External resource] Logique et théorie des ensembles / Dehorny
- Find the references at the Library

Notes/Handbook

Lecture notes available on Moodle (very detailed 417 pages in color!).

Moodle Link

- https://go.epfl.ch/MATH-318