PHYS-755

Introduction to topological phases

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Cursus	Sem.	Type
Photonics		Obl.
Physics		Opt.

Language of teaching	English
Credits	3
Session	
Exam	Oral
Workload	90h
Hours	**57**
Courses	25
Exercises	17
Project	15
Number of positions	**50**

Frequency

Only this year

Summary

This course introduces the elementary topological concepts and tools that recently spread from condensed matter physics to various quantum and classical wave systems.

Content

Lecture 1 : Topology in Physics, a (selective) overview. (1.5h class, week 14)
Topology means many different things, and was used implicitly or explicitly all over the XXth century by physicists, in particular in the study of condensed matter.

Lecture 2 : Elementary notions of topology (toolbox 1) (1.5h class + 1h. homework, week 16)
Euler characteristics, homotopy, topological defects.
Homework: Dislocations

Lecture 3 : Geometrical phases (3h class, week 16)
Gauge invariance. Aharonov-Bohm effect, Dirac magnetic monopoles, Berry phase in quantum systems. Notion of parallel transport.

Lecture 4 : Topology in 1d chains of dimers: SSH models. (3h class + 3h homework, week 16)
First concrete example of a topological model on a lattice. Winding numbers and edge states, bulk-edge correspondence.

Homework: Numerical implementation of the SSH model, spectra. Generalisations (disorder, next hoppings).

Lecture 5 : Topology and degeneracy points (8h classes + 5h homework, week 17)
I) Motivation : Two-level quantum /wave systems
II) toolbox 2 : fiber bundles, differential forms, Stokes theorem, Brouwer theorem.
III) From geometry to topology: 1-form Berry connection, 2-form Berry curvature, semiclassical trajectories of wave packets. 1st Chern numbers, numerical evaluation.
IV) Application : Chern numbers of spin-S models
V) Chern numbers and spectral flows
Quantization and symbol of an operator, analytical index of an operator, application to models in the continuum : 2D massive Dirac fermions, 3D Weyl semimetals in a magnetic field, shallow water model for equatorial waves.

Homework: Analytical expression of the Berry-Chern monopole, computation of spectral flows.

Lecture 6 : Topology and transport properties of band insulators (8h weeks 18 and 20 + 8h homework)
I) Thouless pump
Quasi-adiabaticity, quantized pumped current, time-reversal symmetry breaking, Rice-Mele model.

II) Quantum Hall effect and Chern insulators
Quantized conductivity. Landau levels and Hofstadter butterfly. Anomalous quantum Hall effect, and illustration with the Haldane (lattice) model, quantized Berry phase and Chern numbers, illustration of the bulk-edge correspondence.

Homework: Numerical implementation of the Haldane and/or Rice-Mele models, spectra and Chern numbers.

Lectures : 25h
Homework : 17h (estimated)
Bibliographical project and exam : 15h (estimated), week 22

Note

Pierre Delplace is an invited professor in EPFL STI in 2022. He is only available on specific days, so the entire schedule will be defined and communicated to students in advance (for more details, please e-mail instructors).
Week 16 in room ELA 1 (Buld. ELA)
MON 18.04.22: 9:15-11:00
WED 20.04.22: 9:15-11:00
THU 21.04.22: 9:15-11:00
FRI 22.04.22: 9:15-12:00

Keywords

Berry phase, Chern numbers, bulk-edge-correspondence, spectral flow, quantized conductivity.

Learning Prerequisites

Required courses

None

Learning Outcomes

By the end of the course, the student must be able to:

- To analyze a given physics problem through the lens of topology
- Form a deep understanding of the notion of topological invariants