MATH-643

Applied l-adic cohomology

Michel Philippe

Cursus	Sem.	Type
Mathematics		Opt.

Language of teaching	English
Credits	2
Session	
Exam	Oral
Workload	60h
Hours	**28**
Courses	28
Number of positions	

Frequency

Only this year

Remark

Spring semester

Summary

In this course we will describe in numerous examples how methods from l-adic cohomology as developed by Grothendieck, Deligne and Katz can interact with methods from analytic number theory (prime numbers, modular forms etc...).

Content

In this course we will describe in numerous examples how methods from l-adic cohomology as developed by Grothendieck, Deligne, Katz (and others) interact with methods from analytic number theory (the study of the distribution of prime numbers, size methods or the analytic theory of modular forms and associated L-functions).

Although the major results from l-adic cohomology will be treated as a black box (for instance Deligne's main theorem from « Weil II ») we explain in concrete details how to get ones hands on the « trace function » attached to an l-adic sheaf on the affine line (which is simply the Frobenius trace function of the underlying Galois representation) and how the geometric properties of the sheaf impact its behavior as a function.

Basic properties on traces functions and their associated sheaves:

Sheaves as Galois representations over function fields.
The Frobenius trace functions.
The geometric representation and its invariants: local and global monodromy groups.
The Grothendieck-Ogg-Shafarevich and the Grothendieck-Lefshetz trace formula.
Deligne main theorem from "Weil II" and some of its consequences:
- Correlation of trace functions
- Diophantine criterion for irreducibility

Examples of trace functions and their associated sheaves

Artin-Schreier and Kummer sheaves.
Kloosterman sheaves.
Pull-backs, tensor products, Fourier transform.

Trace functions in everyday life:

-Trace functions and codes.
-Sato-Tate equidistribution laws.
-Trace functions over intervals: the Polya-Vinogradov method and beyond.

-Primes and divisor functions in large arithmetic progressions.
-Trace functions vs. Modular forms.
-Trace functions vs. the primes.

Keywords

Analytic Number Theory, Algebraic Geometry, Etale Cohomology.

Learning Prerequisites

Required courses

Basic Number Theory (algebraic and analytic), Algebraic Geometry

Recommended courses

Modular forms

Learning Outcomes

By the end of the course, the student must be able to:

- Recognize the basic properties of trace functions and how to use these in unconventional settings (outside algebraic geometry)

Resources

Bibliography

N. Katz books at PUP (notably "Gauss Sums Kloosterman Sums and Monodromy"); Arizona Winter School "Lectures on Applied l-adic Cohomology"

Ressources en bibliothèque

- Gauss Sums Kloosterman Sums and Monodromy / Katz
- Lectures on Applied l-adic Cohomology

Websites

- https://arxiv.org/abs/1712.03173