EE-726

Sparse stochastic processes

Unser Michaël

Cursus	Sem.	Type
Electrical Engineering		Opt.

Language of teaching	English
Credits	4
Session	
Exam	Multiple
Workload	120h
Hours	**56**
Lecture	28
Exercises	28
Number of positions	**20**

Frequency

Every 2 years

Remark

Next time: Fall 2024

Summary

We cover the theory and applications of sparse stochastic processes (SSP). SSP are solutions of differential equations driven by non-Gaussian innovations. They admit a parsimonious representation in a wavelet basis and are relevant to coding, compressed sensing, and biomedical imaging.

Content

Mastery of the continuous-domain theory of Gaussian and non-Gaussian stochastic processes and of the corresponding mathematical machinery:

- Generalized functions, (fractional) differential operators, Fourier multipliers, singular integrals
- Innovation models and operator-based resolution of linear stochastic differential equations
- Characteristic form, Lévy-Khinchine representation, functional link with splines, infinite divisibility and sparsity

Representation and analysis of sparse stochastic processes:

- Operator-like wavelets, decoupling using discrete operators, determination of transform-domain statistics

Ability to design algorithms for the recovery of sparse signals with application to biomedical imaging:

- Principled discretization of inverse problems, MAP and MMSE estimators
- Specification of iterative linear reconstruction algorithms using proximal operators and efficient linear solvers

Sparse stochastic processes are continuous-domain processes that admit a parsimonious representation in some matched wavelet-like basis. Such models are relevant to image compression, compressed sensing, and, more generally, to the derivation of statistical algorithms for solving ill-posed inverse problems.
This course is devoted to the study of the broad family of sparse processes that are specified by a generic (non-Gaussian) innovation model or, equivalently, as solutions of linear stochastic differential equations driven by white Lévy noise. It presents the mathematical tools for their characterization. The two leading threads that underly the exposition are:
• the statistical property of infinite divisibility, which induces two distinct types of behavior—Gaussian vs. sparse—at the exclusion of any other
• the structural link between linear stochastic processes and spline functions which is exploited to simplify the mathematics
The concepts are illustrated with the derivation of algorithms for the recovery of sparse signals, with applications to

biomedical image reconstruction. In particular, this leads to a Bayesian reinterpretation of popular sparsity-promoting processing schemes—such as total-variation denoising, LASSO, and wavelet shrinkage—as MAP estimators for specific types of sparse processes. The formulation also suggests alternative recovery procedures that minimize the estimation error.
The course is targeted to an audience of graduate students and researchers with an interest in signal/image processing, compressed sensing, approximation theory, machine learning, or statistics.

For more details, including table of content, see
http://www.sparseprocesses.org/

Keywords

Signal and image processing, sparsity, stochastic modeling, wavelets, compressed sensing.

Learning Prerequisites

Recommended courses

Theory of linear systems, Fourier transform, Signal processing, statistics.

Assessment methods

Midterm and final oral examination.

Resources

Moodle Link

- https://go.epfl.ch/EE-726