

ME-716

**Similarity and Transport Phenomena in Fluid**

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Cursus	Sem.	Type
Mechanics		Opt.

Language of teaching	English
Credits	2
Session	
Exam	Project report
Workload	60h
<b>Hours</b>	<b>28</b>
Lecture	20
Exercises	8
<b>Number of positions</b>	<b>20</b>

**Frequency**

Every 2 years

**Remark**

Next time: Fall 2024

**Summary**

The course is an introduction to symmetry analysis in fluid mechanics. The student will learn how to find similarity and travelling-wave solutions to partial differential equations used in fluid and continuum mechanics. The course covers mathematical and physical aspects

**Content****Chapter 1: The concept of similarity**

- Geometrical similarity
- Invariance by affine transformation, rotation, translation
- Fractal similarity
- Scaling law
- Physical similarity
- Complete similarity: drag force
- Incomplete similarity: flow resistance

**Chapter 2: Transport phenomena in fluid dynamics**

- Transport phenomena
- Advection
- Diffusion Heat equation
- Wave
- Shocks and conservation equations
- Boundary problems: fixed boundary, boundary layer, free boundary problem
- Classification of partial differential equations
- First-order equation, characteristic form
- Second order equation, hyperbolic, elliptic, parabolic

**Chapter 3: One-parameter groups, Lie groups**

- Groups of transformation
- Group invariants
- Invariant curves
- Transformation of derivative

**Chapter 4: First-order differential equations**

- Phase portrait
- Singular point
- Separatrix
- Integrating factor
- Invariant integral curves

- Singular solution
- Change of variables

#### Chapter 5: Second-order differential equations

- Invariant differential equations
- Lie's reduction theorem
- Stretching group
- Singularities

#### Chapter 6: Similarity solutions to partial differential equation

- Similarity solutions
- Associated stretching group
- Asymptotic behavior
- Determining equations

#### Chapter 5: Travelling wave solution

- Translation groups
- Example: diffusion with source
- Propagation velocity
- Approach to travelling waves

#### Chapter 8: Hyperbolic problems

Hyperbolic problems

- One dimensional problems
- Characteristic equations
- Shock formation
- The Riemann problem

Generalization to multidimensional problems

- Linear systems
- Nonlinear systems
- The shallow-water equations

#### Chapter 9: Parabolic problems

- Linear diffusion
- Nonlinear diffusion
- Stefan problem
- Boundary layer problem

### Keywords

partial differential equation, diffusion, advection, similarity solutions, travelling wave solution, hyperbolic problems

### Resources

#### Bibliography

Bibliography is provided on the webpage

#### Ressources en bibliothèque

- [Introduction to Symmetry Analysis / Cantwell](#)
- [Symmetry and Integration Methods for Differential Equations / Bluman](#)
- [Applications of Lie's theory of ordinary and partial differential equations / Dresner](#)
- [Turbulence, coherent structures, dynamical systems and symmetry / Holmes](#)
- [Similarity Solutions of Nonlinear Partial Differential Equations / Dresner](#)
- [Symmetry Methods for Differential Equations -- A Beginner's Guide / Hydon](#)
- [Scaling / Barenblatt](#)
- [Differential Equations: Linear, Nonlinear, Ordinary, Partial / King](#)
- [Application of Lie Groups to Differential Equations / Olver](#)
- [Self-Similarity and Beyond / Sachdev](#)
- [Scaling, Self-Similarity, and Intermediate Asymptotics / Barenblatt](#)

#### Websites

- <http://lhe.epfl.ch/doctorate-en.php>

**Moodle Link**

- <https://go.epfl.ch/ME-716>