MATH-664

Malliavin calculus and normal approximations

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Cursus	Sem.	Type
Mathematics		Opt.

Language of teaching	English
Credits	3
Session	
Exam	Project report
Workload	90h
Hours	**63**
Lecture	21
Exercises	16
Project	26
Number of positions	

Frequency

Only this year

Remark

Fall 2024

Summary

This course will provide a basic knowledge of the stochastic calculus of variations with respect to the Brownian motion. A variety of applications will be presented including the regularity of probability densities and quantitative normal approximations.

Content

This is an introductory course on Malliavin calculus and its applications. The Malliavin calculus is a stochastic calculus of variations with respect to the Brownian motion, that was introduced by Paul Malliavin in the 70's to provide a probabilistic proof of Hörmander's hypoellipticity theorem. The course will cover the main applications of Malliavin calculus including the existence and regularity of probability densities and the rate of convergence in normal approximations via Stein's method. We will also discuss some recent applications to ergodicity and asymptotic behavior of spatial averages of stochastic partial differential equations.
The following topics will be treated:
1. Brownian motion and the Wiener space.
2. An introduction to the stochastic calculus with respect to the Brownian motion and change of variables formulas.
3. Derivative and divergence operators. The divergence operator as a stochastic integral.
4. The Wiener chaos and the Orsntein-Uhlenbeck semigroup. Meyer inequalities.
5. Stochastic integral representations. Clark-Ocone formula.
6. Proving tightness using Malliavin calculus.
7. Density formulas. Criteria for existence and regularity of probability densities and its application to diffusion processes.
8. Stein's method for normal approximations. Rate of convergence in total variation distance.
9. Central limit theorems for stationary sequences.
10. Ergodicity and asymptotic behavior of spatial averages of stochastic partial differential equations.

Keywords

Brownian motion. Stochastic calculus. Diffusion processes. Malliavin calculus. Normal approximations. Stein's method.

Learning Outcomes

By the end of the course, the student must be able to:

- Develop a working knowledge of the techniques of Malliavin calculus with respect to the Brownian motion and must be able to develop further applications.

Resources

Moodle Link

- https://go.epfl.ch/MATH-664